Performance Based Learning and Assessment Task

Basketball Math

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:
The student will be able to understand and apply linear programming techniques in a real world context. Students will be tasked with finding an optimal value given a set of constraints.

II. UNIT AUTHOR:
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III. COURSE:
Algebra, Functions, and Data Analysis

IV. CONTENT STRAND:
Algebra and Functions

V. OBJECTIVES:

- Set up a system of inequalities for the given situation
- Use linear programming to determine the optimal value for the system of inequalities
- Compare predictions to actual results and explain reasons for differences

VI. REFERENCE/RESOURCE MATERIALS:
Students will need a pencil, a graphing calculator, laptops, and the *Basketball Math* worksheet

VII. PRIMARY ASSESSMENT STRATEGIES:
The students will be assessed on their responses on the *Basketball Math* worksheet. The students will be assessed on how clearly they explain their process in the reflection questions on the worksheet.

VIII. EVALUATION CRITERIA:
The students will fill out a self-assessment and the teacher will use the same assessment. A rubric is attached which details each item and how it should be scored on a scale of 0-3. A benchmark is also attached which contains an example of what exemplary work would look like.

IX. INSTRUCTIONAL TIME:
This activity should be completed in 2-3 60 minute class periods. A part of a third period may be needed to wrap up the final discussion and student self-assessments.

Basketball Equations

Strand
Algebra and Functions

Mathematical Objective(s)
The student will be able to write a system of inequalities for the given situation. The student will then be able to use linear programming to find an optimal value. The students will make predictions and compare their predictions to the actual results.

Related SOL
AFDA.5: The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

NCTM Standards
- Interpret representations of functions of two variables
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases;
- Use symbolic algebra to represent and explain mathematical relationships
- Draw reasonable conclusions about a situation being modeled.
- Represent and analyze mathematical situations and structures using algebraic symbols
- Understand patterns, relations, and functions
- Use mathematical models to represent and understand quantitative relationships

Materials/Resources
- Pencil and paper
- Classroom set of graphing calculators
- *Basketball Equations* worksheet
- Laptop cart

Assumption of Prior Knowledge
- Students should understand how to convert percentages to decimals
- Students should understand how to find the percentage of a number
- Students should understand how to write a system of inequalities for a given situation
- Students should understand how to graph a system of inequalities
- Students should understand how to find an optimal value using linear programming techniques
- Students should understand how to use an online graphing calculator on the laptop

Potential difficulties include
- Students may need a refresher on how the online graphing calculator works
- The students may have trouble interpreting the graphs they create
- Task 2 may lead to off-task behavior in the gym without proper monitoring
- Some students may have difficulty making enough baskets in Task 2 to get good data. The teacher may adjust the location of each shot as needed.

Introduction: Setting Up the Mathematical Task

The teacher will set up the task with a discussion about the game of basketball. The teacher may use any relevant storyline from the current NBA or NCAA season to pique the interest of the students. The discussion should cover the point values in basketball and the concept of shooting percentage.

Students will work in pairs to complete Task 1. Students will be given the *Basketball Math* worksheet and will use a system of inequalities to find the optimal score for each player.

The teacher will create groups of 4-5 students for Task 2. The class will go to the gym and shoot baskets to carry out the same game from Task 1. Predictions will be made and compared to the actual results.

Student Exploration

Whole Class discussion:
The teacher may ask the following questions to guide the opening discussion and draw upon the students' prior knowledge:

- How many of you have played basketball?
- How do you score in basketball?
- How much is each basket worth?
    - Use this question to see if students know there are two different point values depending on where the shot is taken (some may even bring up the 1 point free throw, which will not be used in this particular activity).
    - The teacher will be sure the concept of a two point and three point shot is discussed.
- Which shot is typically easier to make? (Students should come to the conclusion that the three pointer is typically more difficult to make than a two pointer. Some students may point out that the difficulty of a two-point shot will vary, depending on how close to the basket the shot is taken.)
- In basketball, how do we determine the success rate of a shot?
    - The teacher will guide this discussion towards the idea of shooting percentage. The teacher can even build on the interest of the students in the room by discussing some of the current NBA stars and their shooting percentages for both two point and three point shots.

Student/Teacher Actions:

- The teacher will hand out the *Basketball Math* sheet to each student.

- Think, Pair, Share Activity:
    - The students will be asked to take a few minutes to analyze the chart and make a prediction as to who will score the most points. Students will be asked to hold off on making any specific calculations and to make their choice based solely on estimation and their initial impressions on the information presented in the table.
    - After a few minutes of individual brainstorming, the students will break off into pairs and discuss their predictions. The pairs will try to come up with a consensus winner. Each pair will then share with the class their predictions and reasoning behind it.
    - The teacher will have a short discussion about expected results vs. actual results. The teacher can ask, "Will the player we expect to win always win?"

- The teacher will guide a quick discussion on how the information in the table is not guaranteed to play out the same every time the game is played. A connection to theoretical and experimental probability can be discussed.
- While the students are in pairs discussing their predictions, the teacher will move around the room from group to group to listen in on what is being said.

- Task 1
  - The teacher will place the students in pairs to complete Task 1 on the *Basketball Math* worksheet
  - One laptop will be given to each pair in order to access an online graphing calculator
  - While the students are completing the first task, the teacher will again move around the room and listen in on the group discussions. The teacher can use the following guiding questions to help the students who are struggling move in the right direction such as:
    - What are the variables?
    - What are the constraints?
    - How can you use inequalities?
    - How do you use the shooting percentages? What form must they be in?
  - Each pair of students will briefly share with the class who they believe will win and why. They must support their decision with logical reasoning.
  - The teacher will then share the solution to the problem and check for understanding informally through a thumbs up, thumbs down response.

- Task 2
  - The teacher will bring the class to the gym and break them into groups of 4-5 students each
  - The students will complete Task 2 of the *Basketball Math* worksheet in their groups.
  - The teacher will move from group to group to be sure the students are on task and on the right track.
  - If students are having a hard time finding their shooting percentage, the teacher may guide them by asking, "How can you calculate your shooting percentage after taking 20 shots?" The teacher can build off of the response from the student and lead them towards dividing the shots made by 20.
  - When a group is finished, the teacher will check for understanding and accuracy. Students from this group may help struggling students if their work is found to be complete and accurate.

Whole Class Wrap-up Discussion

The teacher will lead a final discussion about the overall task. The following questions can be asked to help students make connections between the data and the mathematics involved with predicting a winner.

- *What method did you use to make your prediction in Task 1?* Most students will use a system of inequalities and linear programming but some may use a guess and check method. If a student brings this method up, the teacher can use this as a means to discuss efficiency and using a system of inequalities to allow for repeatability in future instances of this game.
- *What limitations are there to making predictions using this method?* Students may point to the fact that this method allows for partial points to be awarded and in an actual game, this is not possible. For instance a 30% 3-point shooter would be expected to make 1.5 shots out of 5. This is not possible in reality.
- *How closely did your predictions match the actual results in Task 2?*
- *What limitations were there in our procedure for predicting who would win in Task 2?* Students may realize that 20 shots is too small a sample size to determine an accurate shooting percentage that is repeatable. The teacher can follow up this point by asking *"How many shots would provide a proper sample size?"*

Monitoring Student Responses

- Students will be given many opportunities to communicate their thinking throughout this task. The teacher will be leading multiple class discussions and will encourage participation from each student in the class.
- The "think, pair, share" activity will give students the chance to make predictions and then discuss those predictions with a partner. These predications will then be discussed as a class.
- The teacher will move around the room whenever students are working individually or in groups. The teacher will ask guiding questions to make sure the students are zeroing in on the mathematical ideas being investigated.
- It is suggested that the teacher create groups of students based on ability level. The teacher should group students with similar ability levels. This will give the teacher less groups to focus on when offering help. This will also open up the opportunity for a variety of answers to the open-ended questions which leads to further enriching discussion. The higher performing students will typically give the "book answer" but the lower groups may come up with interesting alternatives. Finally, this will allow the high students to help those who are struggling if they finish early.
- This task should conclude with a class discussion. There may not be enough time at the end of day two for this discussion to take place. It is recommended that the teacher take the time on a third day to have a closing discussion and also to have students complete the self-reflection. This closing discussion is summarized above and will allow students to get a good overall understanding of the task that was completed and how it relates to algebra.

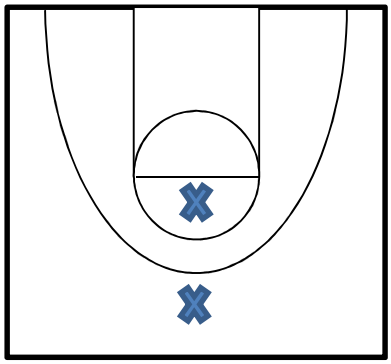
Assessment List and Benchmarks

- The students will each complete Task 1 and Task 2 worksheets
- The students will present their findings after completing Task 1
- The students will complete a self-assessment after completion of both tasks. The teacher will complete this same assessment for each student.
- The assessment worksheets, rubrics, and benchmarks are attached below.

Performance Task- Expressions and Operations Names _____ Date_____

Basketball Math

Task 1: Five friends are playing a game where they can take ten total shots. Each shot must be taken from one of the two designated spots on the court with the following constraints: No more than eight 2-point shots can be taken, no more than five 3-point shots can be taken, and the winning score must be greater than or equal to eight. The person who scores the most points wins the game. Each competitor has a different skill level which is shown by their shooting percentages from each type of shot. The court is set up as shown. The shooting percentages for each competitor is shown in the following table:



Player	2-point %	3-point %
James	50%	30%
Hannah	30%	50%
Janet	40%	25%
Chris	45%	35%
Rachel	60%	15%

a) Who do you expect to win the game? Show all work and give justification for your answer. Show any formulas, tables, equations, etc. that you used.

b) What are some limitations to predicting results using your method? Explain in detail.

Task 2

You are going to play the same game as above with four other classmates. We will go to the gym and collect data. Each student will take 20 shots from each location to determine shooting percentages. You may use the following table to organize your data.

Name	2-point shots made	2-point percentage	3-point shots made	3-point percentage

a) If you play the same game from Task 1, who do you expect to win? Show all work and give justification for your answer. Show any formulas, tables, equations, etc. that you used.

b) Now play the actual game and record the results. Each student determine how many of each shot to take based on their calculations and what will give them the best chance to maximize their score.

Name	Score

c) Compare your predictions with the actual results. Was your method an accurate predictor of who would win? Why or why not?

d) What are some limitations to predicting results using the procedure we used today? What can be changed in order to make more accurate predictions? Explain in detail.

Performance Task- Expressions and Operations Names _____ Date_____

Self-Assessment

			Earned Assessment	
Number	Element	Point Value	Self	Teacher
	Task 1			
1	The student uses an appropriate method to predict the winner	2		
2	The expected winner is correctly identified by the student	2		
3	The student justifies their answer	2		
4	The student shows all work	2		
5	A thorough explanation is given in response to question b	2		
	Task 2			
6	The student correctly calculates the shooting percentages	2		
7	The student uses an appropriate method to predict the winner	2		
8	The expected winner is correctly identified by the student	2		
9	The student justifies their answer	2		
10	The student shows all work	2		
11	The score sheet is completely filled out for each player	2		
12	A thorough explanation is given in response to question c	2		
13	A thorough explanation is given in response to question d	2		
	Totals	26		

Reflection Questions (use the back if necessary)

1. What did I contribute to the group as we worked through this task?

2. What made this task difficult to complete?

3. What did I learn from completing this task?

4. What questions do I still have after completing this task?

5. How can the methods I used today be used in another real world situation?

Performance Task- Expressions and Operations Names _____ Date_____

Category Descriptions: Task 1

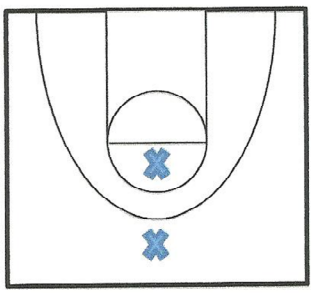
#	Element	0	1	2
1	The student uses an appropriate method to predict the winner	The student did not know what to do	The student used the data to make a prediction but used an incorrect method	The student used an appropriate method
2	The expected winner is correctly identified by the student	No winner is identified	The incorrect winner was identified	The correct winner was identified
3	The student justifies their answer	No justification is given	Does not use complete sentences and/or justification does not make mathematical sense	Justification is in complete sentences and makes mathematical sense
4	The student shows all work	No work is shown	Only partial work is shown	All work is shown
5	A thorough explanation is given in response to question b	No explanation is given	Does not use complete sentences and/or explanation does not make mathematical sense	Explanation is in complete sentences and makes mathematical sense
6	The student correctly calculates the shooting percentages	No values are given	The shooting percentages are partially correct	The shooting percentages are all correct
7	The student uses an appropriate method to predict the winner	The student did not know what to do	The student used the data to make a prediction but used an incorrect method	The student used an appropriate method
8	The expected winner is correctly identified by the student	No winner is identified	The incorrect winner was identified	The correct winner was identified
9	The student justifies their answer	No justification is given	Does not use complete sentences and/or justification does not make mathematical sense	Justification is in complete sentences and makes mathematical sense
10	The student shows all work	No work is shown	Only partial work is shown	All work is shown
11	The score sheet is completely filled out for each player	Score sheet is blank	Score sheet is partially complete	Score sheet is complete
12	A thorough explanation is given in response to question c	No explanation is given	Does not use complete sentences and/or explanation does not make mathematical sense	Explanation is in complete sentences and makes mathematical sense
13	A thorough explanation is given in response to question d	No explanation is given	Does not use complete sentences and/or explanation does not make mathematical sense	Explanation is in complete sentences and makes mathematical sense

Benchmark of Exemplary Student Performance:

Performance Task- Expressions and Operations Names _____ Date_____

Basketball Math

Task 1: Five friends are playing a game where they can take 10 total shots. Each shot must be taken from one of the two designated spots on the court with the following constraints: No more than eight 2-point shots can be taken and no more than five 3-point shots can be taken. The person who scores the most points wins the game. Each competitor has a different skill level which is shown by their shooting percentages from each type of shot. The court is set up as shown. The shooting percentages for each competitor is shown in the following table:



Player	2-point %	3-point %
James	50%	30%
Hannah	30%	50%
Janet	40%	25%
Chris	45%	35%
Rachel	60%	15%

a) Who do you expect to win the game? Show all work and give justification for your answer. Show any formulas, tables, equations, etc. that you used.

Let x = 2-point shots taken
Let y = 3-point shots taken

$x \leq 8$
$y \leq 5$
$x+y \leq 10$
} For all players

James: $(0.5)(2)x + (0.3)(3)y \geq 8$
$x + 0.9y \geq 8$

Hannah: $(0.3)(2)x + (0.5)(3)y \geq 8$
$0.6x + 1.5y \geq 8$

Janet: $(0.4)(2)x + (0.25)(3)y \geq 8$
$0.8x + 0.75y \geq 8$

Rachel: $(0.6)(2)x + (0.15)(3)y \geq 8$
$1.2x + 0.45y \geq 8$

Chris: $(0.45)(2)x + (0.35)(3)y \geq 8$
$0.9x + 1.05y \geq 8$

b) What are some limitations to predicting results using your method? Explain in detail.

First, there is no guarantee that the players will perform up to their prior shooting percentages.

Second, this method allows for partial points to be awarded which is not possible in reality. For example, If Rachel takes three 2-point shots, she is expected to score 3.6 points which is not possible. A game with more total shots would be easier to predict

From Desmos graph:

James' ideal shot selection: 8 2-pointers and 2 3-pointers

Hannah's ideal shot selection: 5 2-pointers and 5 3-pointers

Janet's ideal shot selection: 8 2-pointers and 2 3-pointers

Chris' ideal shot selection: 5 2-pointers and 5 3-pointers

Rachel's ideal shot selection: 8 2-pointers and 2 3-pointers

James' optimal score: 9.8

Hannah's optimal score: 10.5

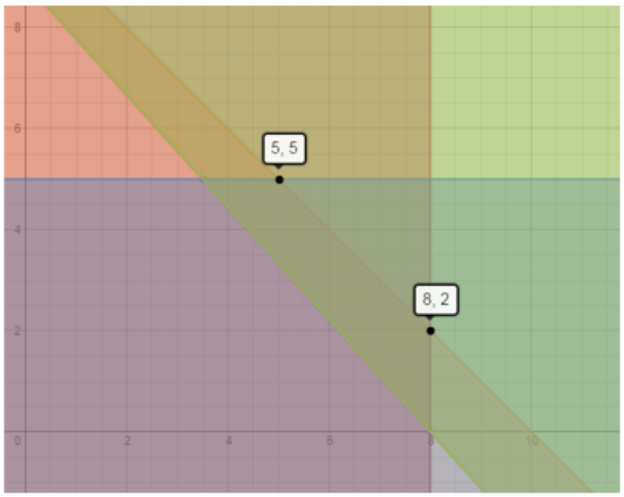
Janet's optimal score: 7.9

Chris' optimal score: 9.75

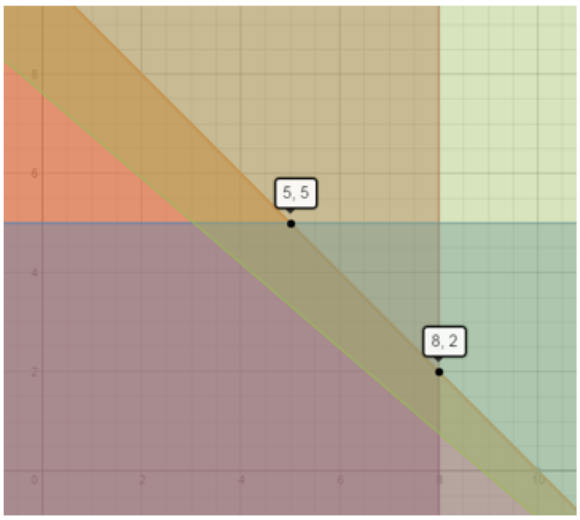
Rachel's optimal score: 10.5

Hannah and Rachel project to have the highest scores. I expect Rachel to win the game because I believe taking more 2-point shots will give her the advantage in the long run. Though Hannah has a 50% shooting percentage, she will probably have a harder time matching that than Rachel will matching the 60% from 2-point range.

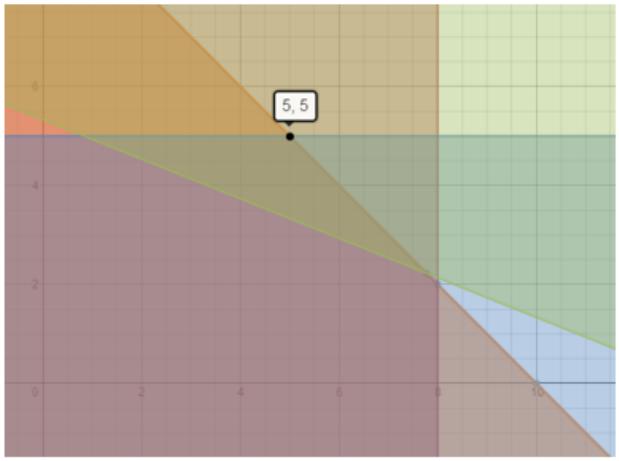
James: Both these points work, (8,2) is the optimal



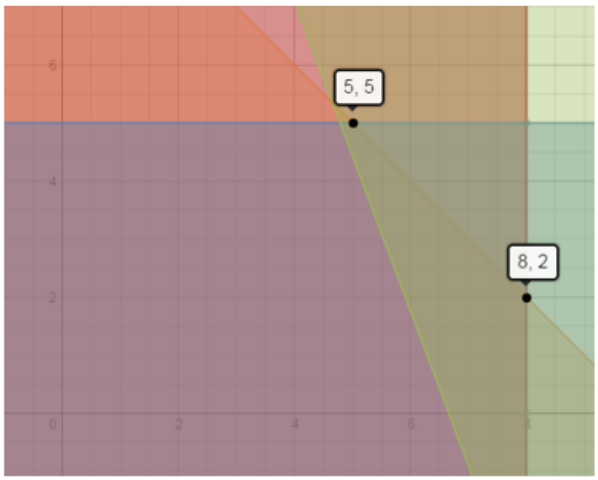
Chris: Both of these points work, (5,5) is the optimal



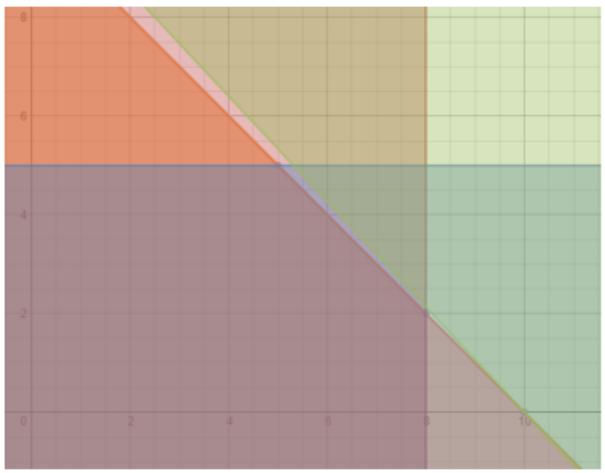
Hannah: (5,5) is the optimal



Rachel: Both of these points work, (8,2) is optimal



Janet: Cannot reach 8 points



Task 2

You are going to play the same game as above with four other classmates. We will go to the gym and collect data. Each student will take 20 shots from each location to determine shooting percentages. You may use the following table to organize your data.

Name	2-point shots made)	2-point percentage	3-point shots made)	3-point percentage
Player 1	8	40%	4	20%
Player 2	6	30%	6	30%
Player 3	10	50%	5	25%
Player 4	4	20%	6	30%
Player 5	7	35%	3	15%

a) If you play the same game from Task 1, who do you expect to win? Show all work and give justification for your answer. Show any formulas, tables, equations, etc. that you used.

x = 2-pointers taken y = 3-pointers taken

Player 1: $0.8x + 0.6y \geq 8$

Player 2: $0.6x + 0.9y \geq 8$

Player 3: $x + 0.75y \geq 8$

Player 4: $0.4x + 0.9y \geq 8$

Player 5: $0.7x + 0.45y \geq 8$

All Players

$x \leq 8$

$y \leq 5$

$x + y \leq 10$

Optimal Scores

Player 1: 7.6

Player 2: 7.5

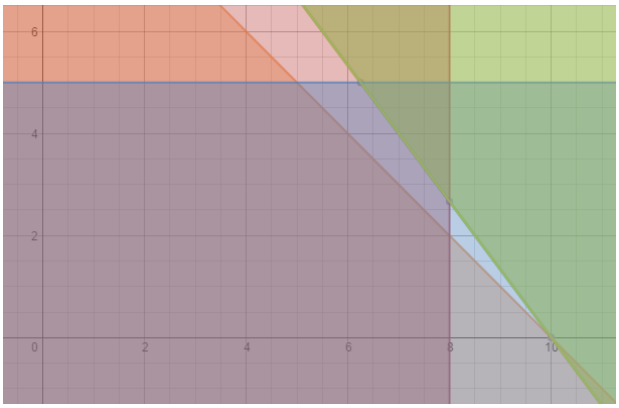
Player 3: 9.5

Player 4: 6.5

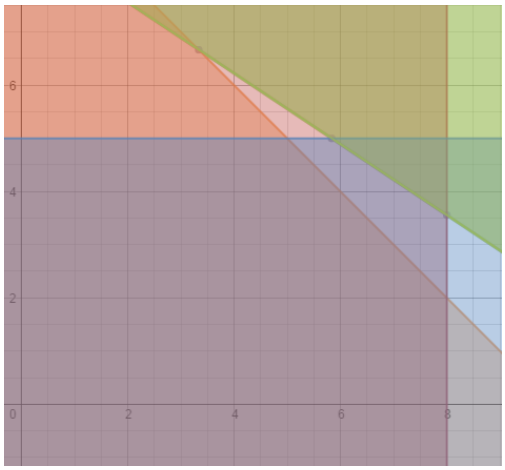
player 5: 6.5

Player 3 is expected to win and is the only player expected to score over the required 8 points.

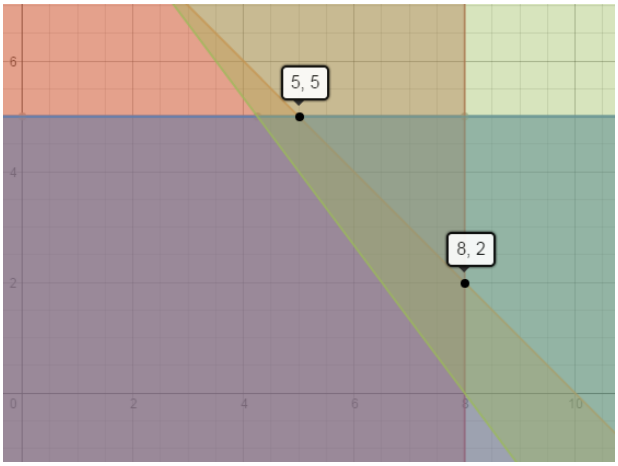
Player 1: Cannot reach 8 points



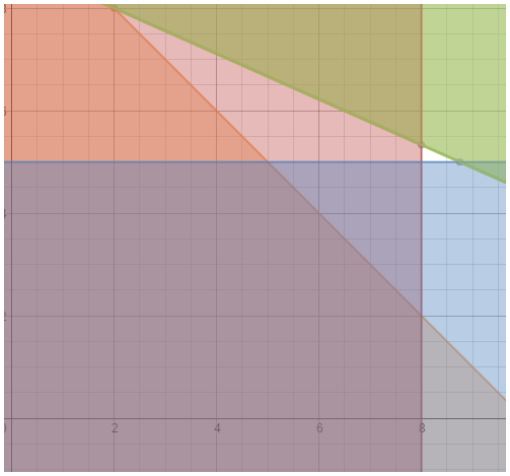
Player 2: Cannot reach 8 points



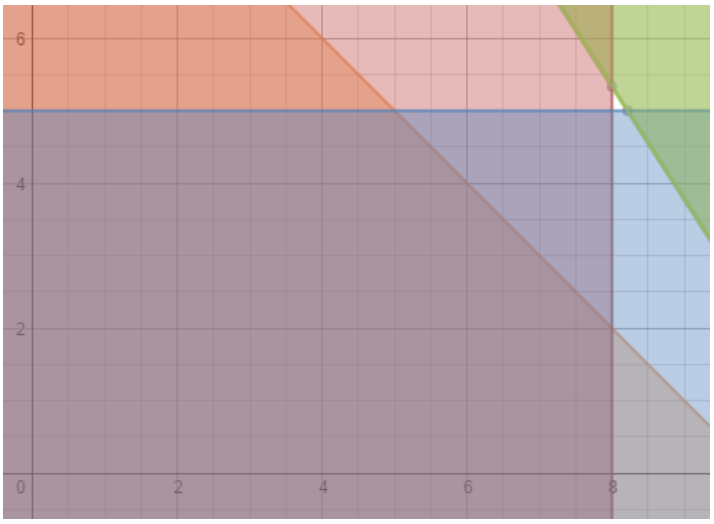
Player 3: Both points work, (8,2) is optimal



Player 4: Cannot reach 8 points



Player 5: Cannot reach 8 points



b) Now play the actual game and record the results. Each student determine how many of each shot to take based on their calculations and what will give them the best chance to maximize their score.

Name	Score

These will vary from group to group

c) Compare your predictions with the actual results. Was your method an accurate predictor of who would win? Why or why not?

These answers will vary. Most likely, the predictions will not match the actual results. The main reason for this is the performance not matching the shooting percentages.

d) What are some limitations to predicting results using the procedure we used today? What can be changed in order to make more accurate predictions? Explain in detail.

The sample size used to come up with the shooting percentages is too small. If time allowed, each student could shoot 100 shots from each spot. This would lead to more accurate predictions. The same issue with partial points being awarded comes up again.